6.6 Factoring: A General Strategy

Objectives

1) Use the factoring process to decide which factoring technique to use
2) Solve equations by factoring

Math 70: 6.6 Solving Equations by Factoring

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1) Choose the correct factoring process.
2) Factor by substitution.
3) Factor expressions containing rational exponents
4) Solve equations by factoring

Examples

1) Solve $2x^3 = 50x$

2) Solve $y^2 + (y+2)^2 = 34$

3) Find the x-intercepts of $f(x) = (x+4)(5x-1)$

Math 70: 6.6 Solving Equations by Factoring

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Math 70: 6.6 Factoring Process, Rational Exponents, Factoring by Substitution

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Review of chapter 5: Multiply

1) $x^{1/3}(2x-5)$

2) $\left(3x^{1/4}+5\right)\left(3x^{1/4}-5\right)$

Factor completely.

3) $x^{2/7}-3x^{1/7}$

4) $3x^{-3/5}-3x^{2/5}$

5) $2(a+3)^2+5(a+3)-7$

6) $5x^{-6}+29x^{-3}-42$

7) $5x^{2/3}+29x^{1/3}-42$

8) $5t^4-80$

9) $6x^2y^4-21x^3y^5+3x^2y^6$

10) x^6-64

11) $-25m^2-20mn-4n^2$

12) $x^2y^2+7xy+12$

13) $2a^3+12a^2+18a-8ab^2$

Math 70 5.5 Factoring
5.6
5.7

Process for Factoring

Step 0: Arrange the terms in standard form, descending from the leading (highest-degree) term first. If there is more than one variable, choose a variable and arrange in descending order by that variable.)

Step 1: Factor out the greatest common factor from all terms.

Step 2: Count the terms.
(Terms are separated by add or subtract symbols, except when the addition or subtraction symbol is already inside parentheses.)

Step 3: If you have 2 terms, factor it by its pattern:

3a: Sum of squares: $a^2 + b^2$ is prime.

3b: Difference of squares: $a^2 - b^2 = (a-b)(a+b)$

3c: Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

3d: Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

[In 3c and 3d the acronym SOAP can be used to remember the three signs in the factors: Same – Opposite – Always Positive]

Step 4: If you have 3 terms, determine which of the following applies:

4a: Perfect Square Trinomial (sum): $a^2 + 2ab + b^2 = (a+b)^2$

4b: Perfect Square Trinomial (difference): $a^2 - 2ab + b^2 = (a-b)^2$

4c: Leading coefficient 1: $x^2 + bx + c$; Find two numbers that multiply to c and add to b using guess-and-check or magic X.

4d: Leading coefficient not 1: $ax^2 + bx + c$;

Use guess-and-check by finding numbers for the first terms that multiply to a and numbers for the second terms that multiply to c, or

Use the "double magic X" by finding two numbers that multiply to the product ac and add to b, then use these to rewrite the middle term and factor by grouping.

4e: If the expression is quadratic in form, $a(garbage)^2 + b(garbage) + c$,

Substitute u=*garbage* to get a true quadratic, factor using u and one of the methods 4a-4d, then replace u by *garbage*, simplify inside parentheses. Check for greatest common factor.

Step 5: If you have 4 terms, factor by grouping.

4a: Two groups of two terms results in two binomial factors.

4b: [Less common]: group three terms to make a perfect square trinomial minus a constant, then factor as a difference of squares.

Step 6: Check each factor to see if it can be factored. Continue factoring until every factor is prime. When you are done, you should have one term with all add and subtract signs inside parentheses.)

Step 7: When in doubt, multiply your result. You should get your original expression (or a simplified version of it). Factoring is the opposite of multiplying.

Math 70

6.6 Solving Equations by Factoring

Zero-product property

if $(a)\cdot(b)=0$ then $(a)=0$
or $(b)=0$.

Note: The zero is essential!

if $(a)\cdot(b)=1$ then $a=\frac{1}{b}$, but there are infinitely many values of b we could substitute!

Solve

*final exam!

① $2x^3=50x$

step 1: set equal to zero so we can use the zero-product property

$2x^3-50x=0$

$2x(x^2-25)=0$ GCF $2x$

step 2: Factor completely

$2x(x-5)(x+5)=0$ diff of squares

step 3: set each factor $=0$

$2x=0$ $x-5=0$ $x+5=0$

$\boxed{x=0}$ $\boxed{x=5}$ $\boxed{x=-5}$

step 4: isolate variable in each result

② Solve $y^2+(y+2)^2=34$

$\underbrace{y^2}_{\text{1st term}}+\underbrace{(y+2)^2}_{\text{2nd term}}\underbrace{-34}_{\text{3rd term}}=0$

set $=0$
But it's not factored; there are 3 terms.

FOIL $(y+2)^2$ and start over to factor

$y^2+(y+2)(y+2)-34=0$

$y^2+y^2+4y+4-34=0$

Math 70

$2y^2 + 4y - 30 = 0$

combine like terms

option 1: divide both sides of equation by 2.

$$\frac{2y^2}{2} + \frac{4y}{2} - \frac{30}{2} = \frac{0}{2}$$

$$y^2 + 2y - 15 = 0$$

-15
$5 \quad -3$
2

$$(y+5)(y-3) = 0$$

$\boxed{y = -5}$ $\boxed{y = 3}$

option 2: Factor out GCF $= 2$.

$$2(y^2 + 2y - 15) = 0$$

$$2(y+5)(y-3) = 0$$

$2 \neq 0$ no soln from this factor

$\boxed{y = -5}$ $\boxed{y = 3}$

③ Find the x-intercepts of $f(x) = (x+4)(5x-1)$

x-int: a point on the x-axis, where the graph crosses the x-axis

$\Rightarrow$ y-coordinate must be zero.

$\Rightarrow$ replace $y = f(x)$ by 0:

$$0 = (x+4)(5x-1)$$

It's already factored!

$x + 4 = 0$ $\qquad$ $5x - 1 = 0$

$\boxed{x = -4}$ $\qquad$ $\boxed{x = \frac{1}{5}}$

Math 70

Multiply

① $x^{1/3}(2x-5)$ distribute

$= x^{1/3}\cdot 2x - x^{1/3}\cdot 5$ add exponents

$= 2x^{1/3+1} - 5x^{1/3}$ $\frac{1}{3}+1 = \frac{1}{3}+\frac{3}{3} = \frac{4}{3}$

$= \boxed{2x^{4/3} - 5x^{1/3}}$

② $(3x^{1/4}+5)(3x^{1/4}-5)$ FOIL

$= 9x^{1/4+1/4} - 15x^{1/4} + 15x^{1/4} - 25$

$= \boxed{9x^{1/2} - 25}$

Factor completely.

③ $x^{2/7} - 3x^{1/7}$

$= x^{1/7}(x^{2/7-1/7} - 3)$ GCF: lowest exp $\frac{1}{7} < \frac{2}{7}$

GCF $x^{1/7}$

$= \boxed{x^{1/7}(x^{1/7}-3)}$

④ $3x^{-3/5} - 3x^{2/5}$ GCF: lowest exp $-3/5 < 2/5$

GCF $3x^{-3/5}$

$= 3x^{-3/5}(1 - x^{2/5-(-3/5)})$

$= 3x^{-3/5}(1 - x^{2/5+3/5})$

$= \boxed{3x^{-3/5}(1-x)}$

⑤ $\underbrace{2(a+3)^2}_{\text{1st term}} + \underbrace{5(a+3)}_{\text{2nd term}} - \underbrace{7}_{\text{3rd term}}$

Factor by substitution

$u = a+3 \Rightarrow u^2 = (a+3)^2$

$= 2u^2 + 5u - 7$

-14

$7 \quad -2$

5

$\rightarrow 2u^2 + 7u - 2u - 7$

$= u(2u+7) - 1(2u+7)$

$= (2u+7)(u-1)$

$= (2(a+3)+7)((a+3)-1)$ replace $u=a+3$

dist 2.

cont...

Math 70

$= (2a+6+7)(a+3-1)$ combine like terms

$= \boxed{(2a+13)(a+2)}$

(6) $5x^{-6} + 29x^{-3} - 42$

factor by substitution
$u = x^{-3} \Rightarrow u^2 = (x^{-3})^2 = x^{-6}$

$\rightarrow 5u^2 + 29u - 42$

-210

29

-1, 210
-2, 105
-3, 70
-5, 42
-6, 35 $\rightarrow$ 29 ✓

$= \underbrace{5u^2 - 6u} + \underbrace{35u - 42}$ grouping

$= u(5u-6) + 7(5u-6)$

$= (5u-6)(u+7)$ replace $u = x^{-3}$

$= \boxed{(5x^{-3}-6)(x^{-3}+7)}$

(7) $5x^{2/3} + 29x^{1/3} - 42$

factor by substitution
$u = x^{1/3}$ $du = (x^{1/3})^2 = x^{2/3}$

$5u^2 + 29u - 42$
(same as (6))

$(5u-6)(u+7)$ replace $u = x^{1/3}$

$\boxed{(5x^{1/3}-6)(x^{1/3}+7)}$

(8) $5t^4 - 80$ GCF 5

$= 5(t^4 - 16)$ diff of squares

$= 5(t^2-4)(t^2+4)$

another diff of squares

sum of squares is prime

$= \boxed{5(t-2)(t+2)(t^2+4)}$

Math 70

(9) $6x^2y^4 - 21x^3y^5 + 3x^2y^6$

GCF $3x^2y^4$

$= \boxed{3x^2y^4(2 - 7xy + y^2)}$

no x^2 here, can't factor further

(10) $x^6 - 64$

diff of squares

$\sqrt{x^6} = x^3 \quad \sqrt{64} = 8$

$= (x^3 - 8)(x^3 + 8)$

diff of cubes and sum of cubes

$= \boxed{(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)}$

(11) $-25m^2 - 20mn - 4n^2$

GCF $-$

$= -(25m^2 + 20mn + 4n^2)$

$(5m)^2$ $\quad 2(5m)(2n)$ ✓ $\quad (2n)^2$

a^2, b^2
see squares in first and last terms $\longrightarrow$ check middle term for $2ab$

$= \boxed{-(5m+2n)^2}$

(12) $x^2y^2 + 7xy + 12$

$\boxed{(xy+3)(xy+4)}$

12, 3, 4, 7

or use X approach

$x^2y^2 + 3xy + 4xy + 12$

$xy(xy + 3) + 4(xy + 3)$

$\boxed{(xy+4)(xy+3)}$

(13) $2a^3 + 12a^2 + 18a - 8ab^2$

GCF $2a$

$= 2a(\underbrace{a^2 + 6a + 9}_{\text{perfect square trinomial}} - 4b^2)$

$a^2 \quad 2a{\cdot}3 \quad 3^2$

$= 2a((a+3)^2 - 4b^2)$

diff of sq. $A^2 - B^2$ $\quad A = (a+3)$, $B = 2b$.

$= 2a[(a+3) - 2b][(a+3) + 2b]$

$= \boxed{2a(a+3-2b)(a+3+2b)}$